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Math 118 - Spring 2022 - Common Final Exam, version A

Print name:____

Section number:__________ **Instructor's name:**____________________________

Directions:

- This exam has 12 questions. Please check that your exam is complete, but otherwise keep this page closed until the start of the exam is called.
- Fill in your name, and your instructor's name.
- It will be graded out of 122 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- A formula sheet has been provided with this exam. You may not refer to any other notes during the exam.
- You may use a calculator which does not allow internet access. The use of any notes or electronic devices other than a calculator is prohibited.

Good luck!

Question:	1	2	3	4	5	6	7
Points:	12	10	10	10	10	14	10
Score:							

Question:	8	9	10	11	12		Total
Points:	10	8	10	10	8		122
Score:							

1. In 2015 the number of people infected by a virus was $P_0 = 325000$. Due to a new vaccine, the number of infected people has decreased by 25% each year since 2015. In other words, only 75% as many people are infected each year as were infected the year before.

   (a) (4 points) Find a formula for the function $P(t)$, the number of infected people t years after 2015 in the form $P(t) = P_0 \cdot b^t$.

   (b) (4 points) Write the formula for the function $P(t)$ in the form $P(t) = P_0 \cdot e^{kt}$. Round constants in your answer to four decimal places.

   (c) (4 points) Find the number of people infected by the virus in 2025. Round your answer to the nearest integer.

2. The tables below contain values from an exponential or a linear function. In each part:

- Decide if the function is linear or exponential.
- Find a formula for the function.

(a) (5 points)

x	0	1	2	3	4
$f(x)$	10.3	11.124	12.01392	12.975	14.013

(b) (5 points)

x	0	1	2	3	4
$i(x)$	30	28	26	24	22

3. (10 points) An investment decreases by 68% over a 14-year period. At what effective annual percent rate does it decrease? Give your answer as a percentage rounded to two decimal places, like $nn.nn\%$.

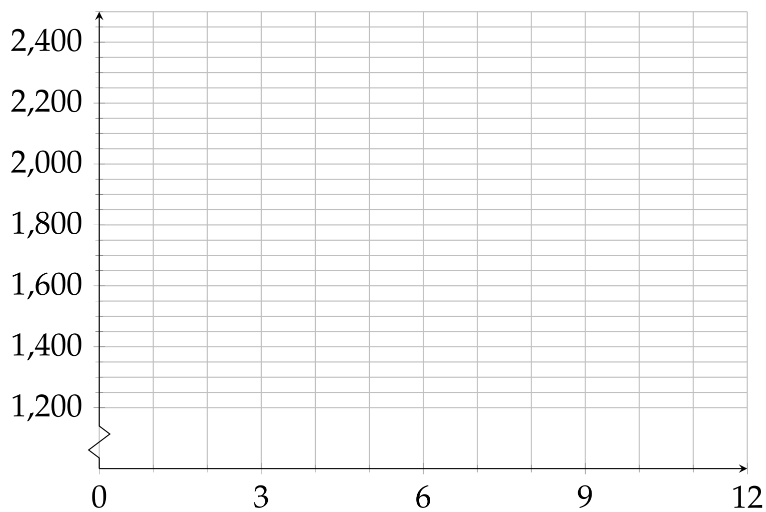
4. (10 points) Which would earn more money: $\$30,000$ invested in an account A paying 2% compounded daily (365 times per year), or the same $\$30,000$ invested in an account B paying 2.1% compounded monthly (12 times per year)? In both cases, suppose the investment lasts for 15 years.

5. (10 points) Find the half-life of a radioactive substance that decays at a continuous rate of 6% per minute. Round your answer to two decimal places.

6. A population of animals oscillates sinusodally between a high of 2400 on January 1 $(t = 0)$ and a low of 1800 on April 1 $(t = 3)$.

(a) (6 points) Find a formula for the population, P, in terms of the time, t, in months.

(b) (4 points) Graph P as a function of t. Mark two points on your graph where $P = 2000$.



(c) (4 points) Now use the formula for the population from part (a) to find when the population first reaches 2,000. Give your answer in terms of an inverse trig function, and give your answer rounded to three decimal places with correct units.

Exam continues...

7. (10 points) For $\frac{\pi}{2} \leq \theta \leq \pi$ find $\tan(\theta)$ if $\cos(\theta) = -\frac{\sqrt{2}}{3}$. Give the exact answer.

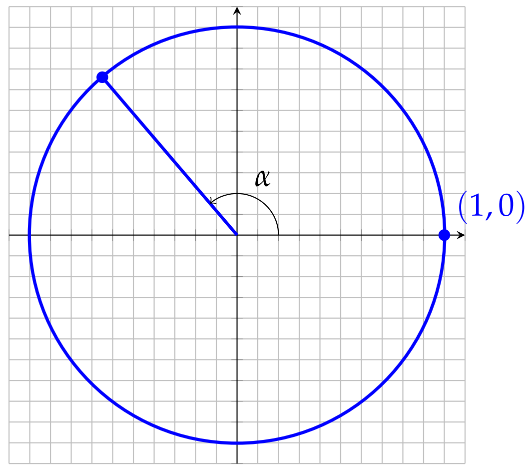
8. Let $P = f(t) = 14 \cdot 2^{t/12}$ give the size in thousands of an animal population in year t.

   (a) (5 points) Find the inverse function $f^{-1}(P)$.

   (b) (5 points) Evaluate $f^{-1}(24)$. Round your answer to two decimal places. In a sentence with correct units, explain what this tells you about the animal population.

Exam continues...

9. The diagram below shows a unit circle (with radius 1). Grid lines are spaced $\frac{1}{10}$ unit apart. The radius corresponding to angle α is shown.

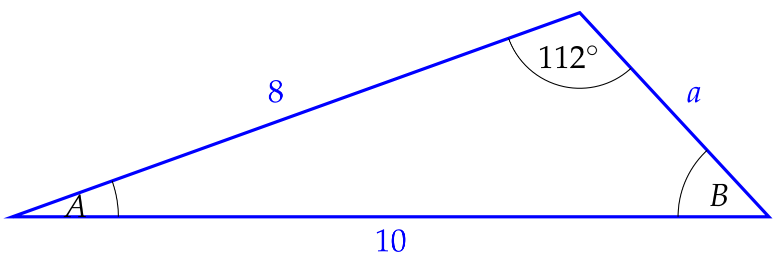


(a) (4 points) Using the grid, estimate $\cos(\alpha)$ and $\sin(\alpha)$ to two decimal places.

- $\cos(\alpha) \approx$ ____________
- $\sin(\alpha) \approx$ ____________

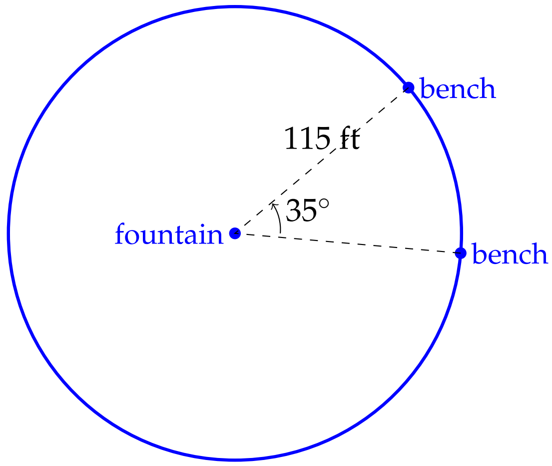
(b) (4 points) Draw another radius on the circle corresponding to an angle in Quadrant III with the same cosine as α.

10. (10 points) Find the missing side and angle measures in the diagram below. Give angles as degrees, and round all answers to one decimal place.



Exam continues...

11. The diagram below shows the location of two benches at the edge of a circular plaza with a fountain in the middle.

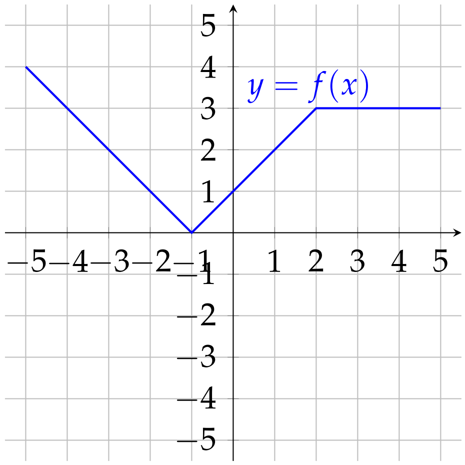


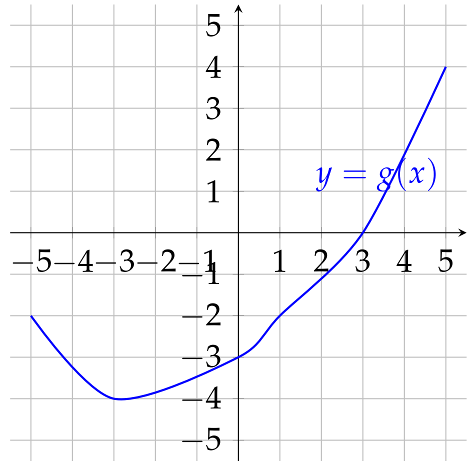
(a) (4 points) The central angle between the benches is 35°. Give this angle in radians, with an exact answer in terms of π.

(b) (6 points) The radius of the circular path is 115 feet. Find the length of the short arc between the benches. Give an answer rounded to two decimal places with correct units.

Exam continues...

12. The graphs of $y = f(x)$ and $y = g(x)$ are given below. Use them to evaluate the following compositions.





(a) (4 points) $f\big(g(-3)\big)$

(b) (4 points) Find all x solving $g\big(f(x)\big) = -2$.

End of exam

Exponential and Logarithm Formulas

Exponential Function: $y = ab^x$

Simple Interest: $P(t) = P_0(1+r)^t$

Compound Interest: $P(t) = P_0(1+\frac{r}{n})^{nt}$

Continuous Growth: $P(t) = P_0e^{rt}$

Half-life: $Q(t) = Q_0(\frac{1}{2})^{\frac{t}{T_h}}$

Doubling time: $Q(t) = Q_0 2^{\frac{t}{T_d}}$

Logarithms: $b^x = M \Leftrightarrow \log_b(M) = x$

Natural Logarithm: $\ln(x) = \log_e(x)$

Common Logartithm: $\log(x) = \log_{10}(x)$

Trigonometry

1 radian $= \frac{180}{\pi}$ degrees

1 degree $= \frac{\pi}{180}$ radians

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} \qquad \csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\cos(\theta) = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} \qquad \sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\tan(\theta) = \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

$$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{\cos(\theta)}{\sin(\theta)}$$

Pythagorean Identity:

$\sin^2(\theta) + \cos^2(\theta) = 1$

$\tan^2(\theta) + 1 = \sec^2(\theta) \qquad 1 + \cot^2(\theta) = \csc^2(\theta)$

Arc Length: $s = r\theta$

Sinusoidal Functions:

$f(x) = A\sin(Bx) + k \qquad g(x) = A\cos(Bx) + k$

Period: $P = \dfrac{2\pi}{B}$

Even-Odd Identities:

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

Other Identities:

$\sin(\theta) = \sin(180^\circ - \theta)$

$\cos(\theta) = -\cos(180^\circ - \theta)$

$\tan(\theta) = -\tan(180^\circ - \theta)$

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos(C)$

Law of Sines: $\dfrac{\sin(A)}{a} = \dfrac{\sin(B)}{b} = \dfrac{\sin(C)}{c}$

Inverse Trig:

$\theta = \cos^{-1} y$ provided that $y = \cos\theta$ and $0 \le \theta \le \pi$.

$\theta = \sin^{-1} y$ provided that $y = \sin\theta$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

$\theta = \tan^{-1} y$ provided that $y = \tan\theta$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

Sum and Difference:

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Double-Angle:

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$
$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$